Making a hash of things

Looking at cryptographic hash functions

March 2006 kw.pm

`http://kw.pm.org/`

Overview

Hash functions

- Some definitions
- Cryptographic requirements
- Some applications
- Generic attacks
  - Optimizing generic attacks
  - Finding meaningful collisions
- Iterated hash functions
  - MD5
  - SHA-1
- Signatures
  - Forging signatures

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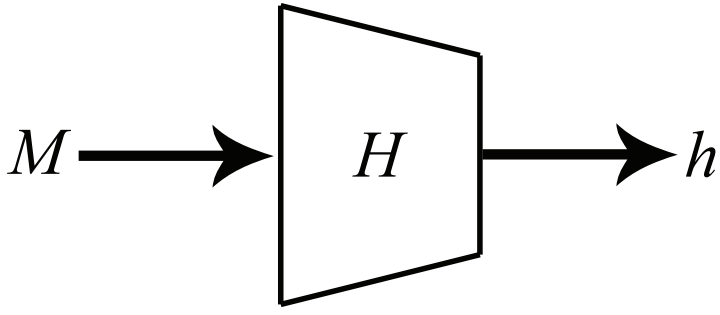
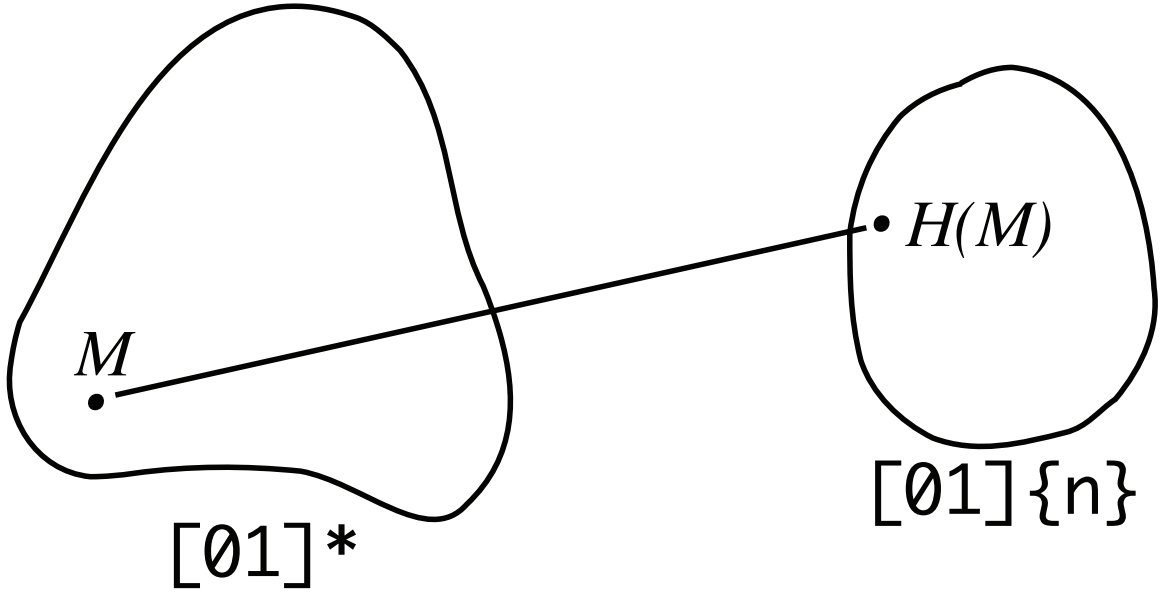
That is not a small number!

Hash functions



Simply put, a hash function takes some input of variable length, and returns a value of fixed length.

Hash space



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- Collision resistance
  How hard is it to find to two arbitrary messages, $M_1, M_2, M_1 \neq M_2$ that have the samd hash value, $H(M_1) = H(M_2)$.

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- Digital signatures. Requires all three resistances.

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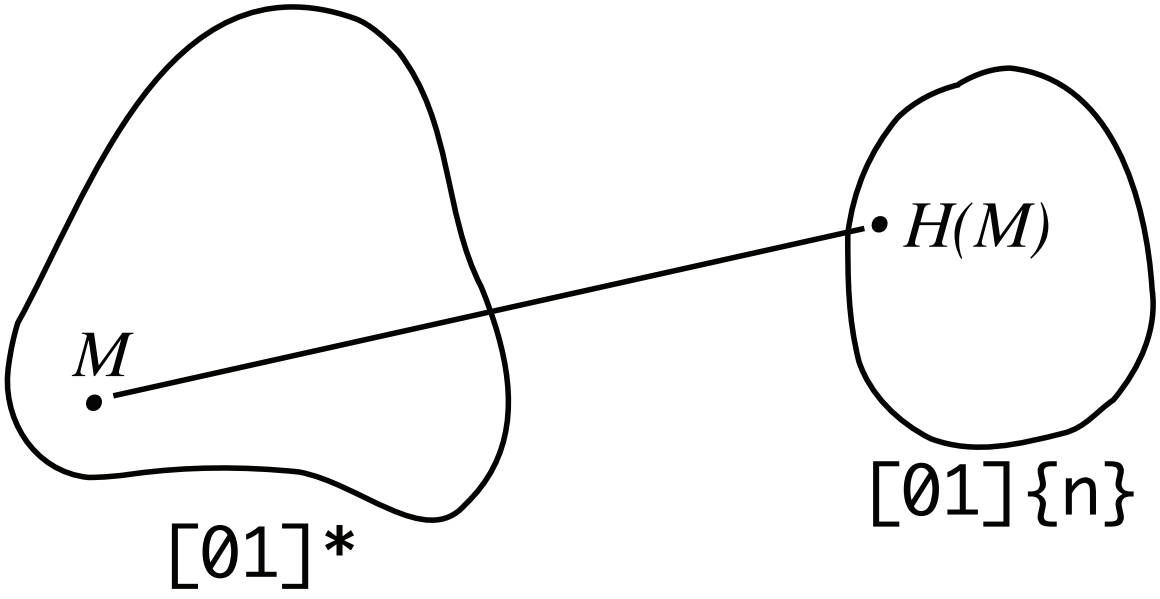
- Finding preimages and 2nd preimages
  - There is no efficient way to find these with a generic attack.
  - Simply continue to pick arbitrary x until $H(x) = y$ or $H(x) = H(x_0)$

Finding collisions



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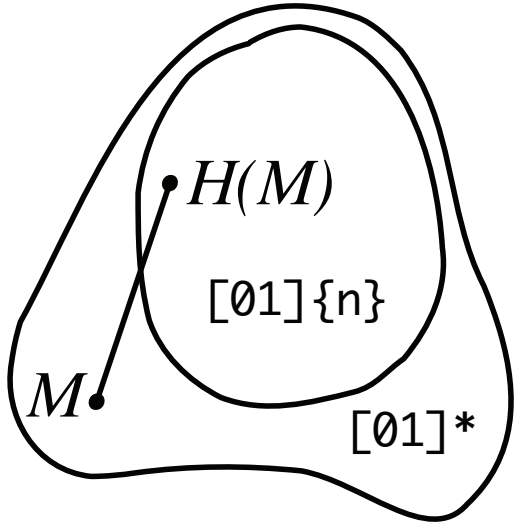

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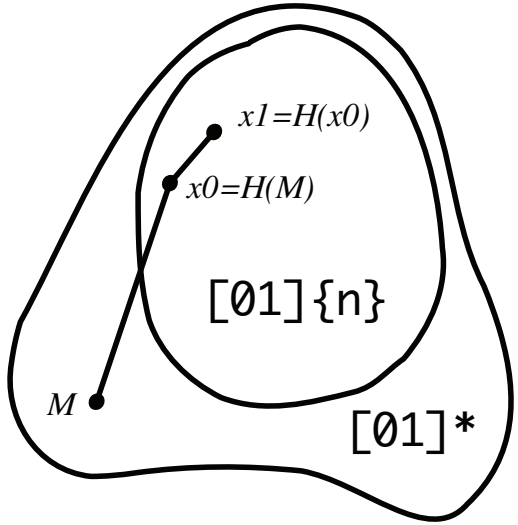

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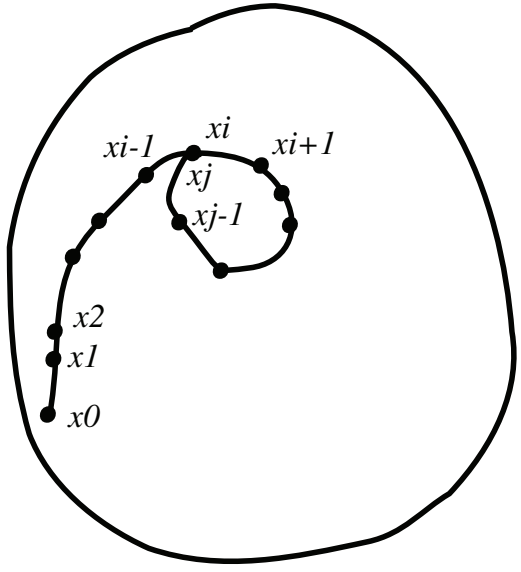

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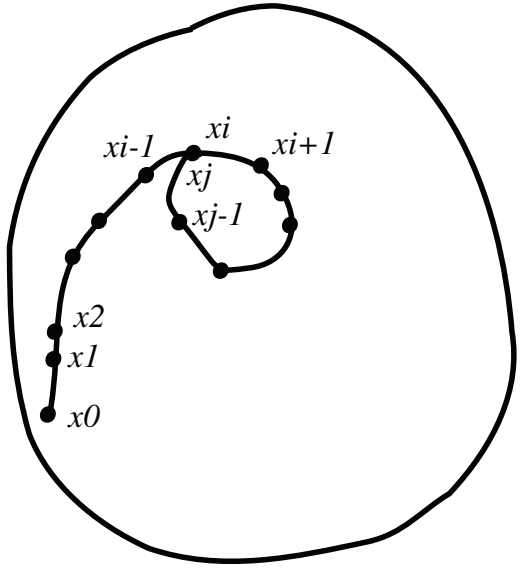

Expected running time:

$$j = \sqrt{\frac{\pi}{2} 2^n}, i = \frac{j}{2}$$

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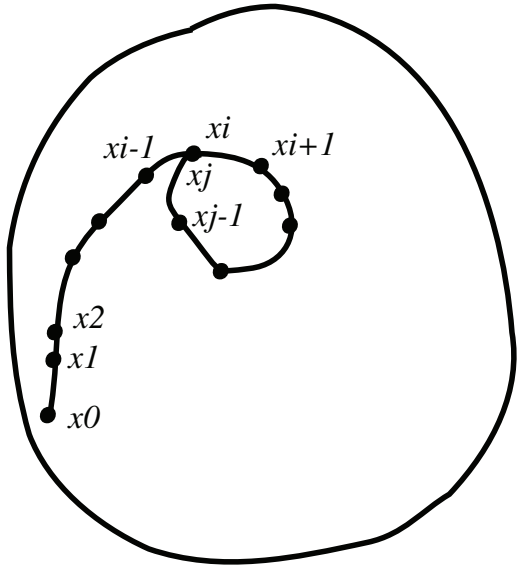
- However, implementing this algorithm would require a hash table containing 2^{64} entries.
  This requires storage in the order of 10^8 terabytes!

A better approach to finding collisions

In 1993, Van Oorschot & Wiener discovered an attack that had approximately the same running time, but negligible space requirements.



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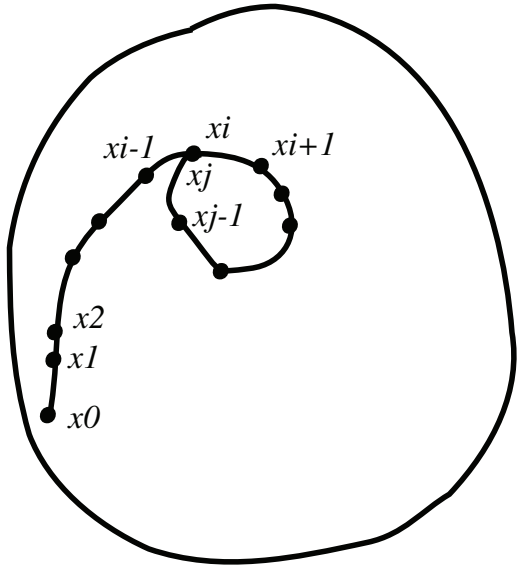

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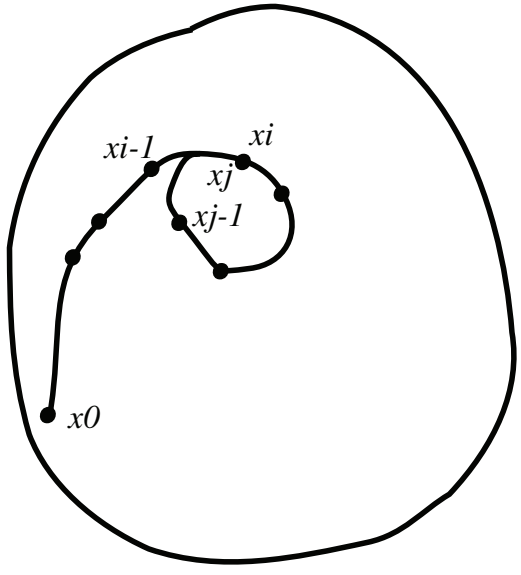

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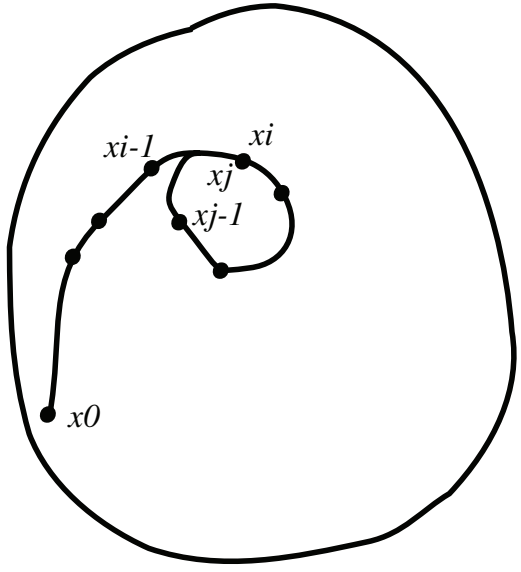

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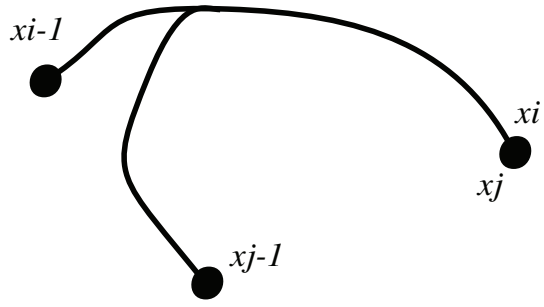

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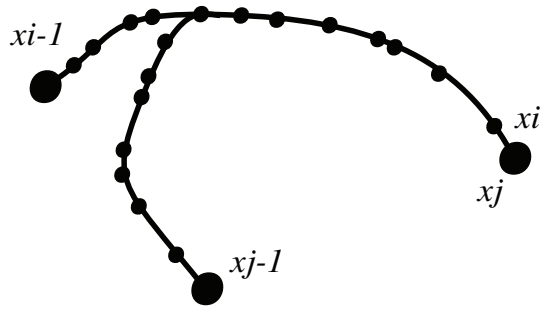

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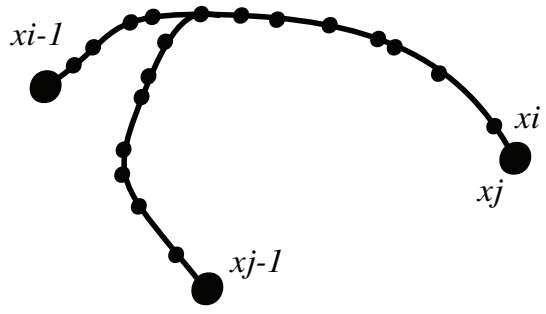

$$\sigma = 32, j = \sqrt{\frac{\pi}{2} 2^n} \approx 2^{64}$$

$$j' = \frac{j}{2^\sigma} \approx 2^{32}$$

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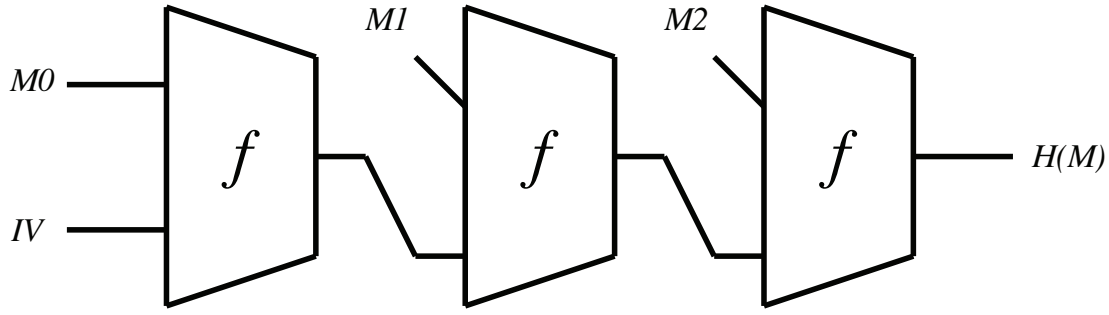

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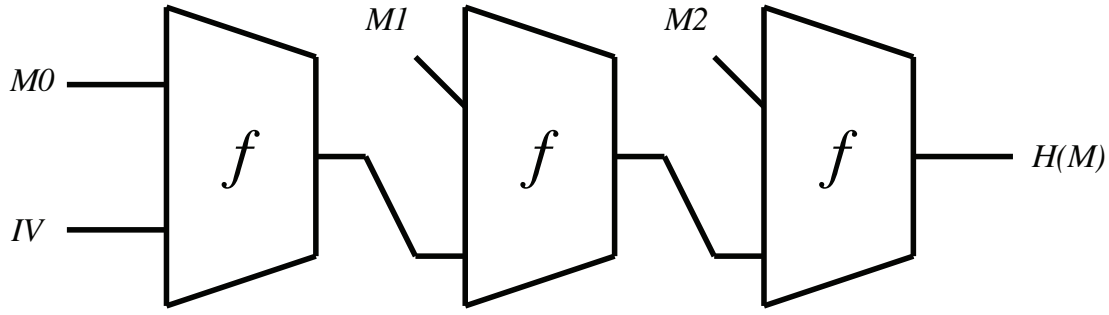

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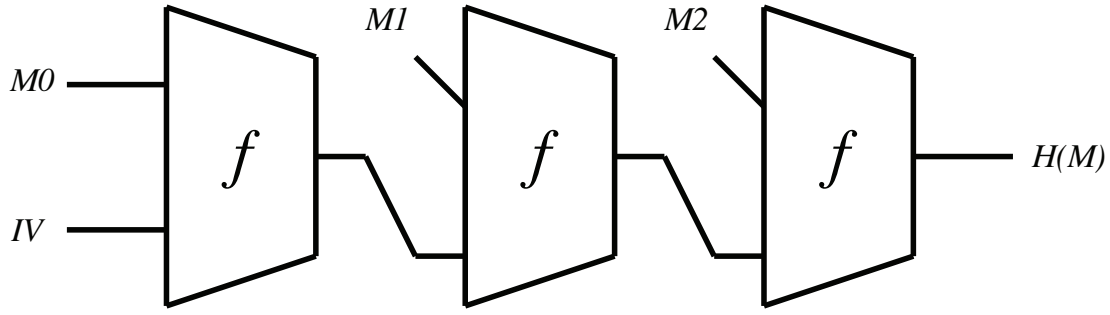

- Messages are broken up into blocks and fed in to successive iterations along with the output from the previous iteration
- **MD5** (128 bits) and **SHA-1** (160 bits) are both examples of this type of hash function.
- More effecient algorithms of finding collisions have been found for both functions.

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- . . . or useful versions of existing messages.

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- Define a new hash function as a composite of this function and your existing hash function,

$$H'(x) = \begin{array}{ll} H(G_1(\frac{x-1}{2})) & \text{if } x \text{ is odd} \\ H(G_2(\frac{x}{2})) & \text{if } x \text{ is even} \end{array}$$

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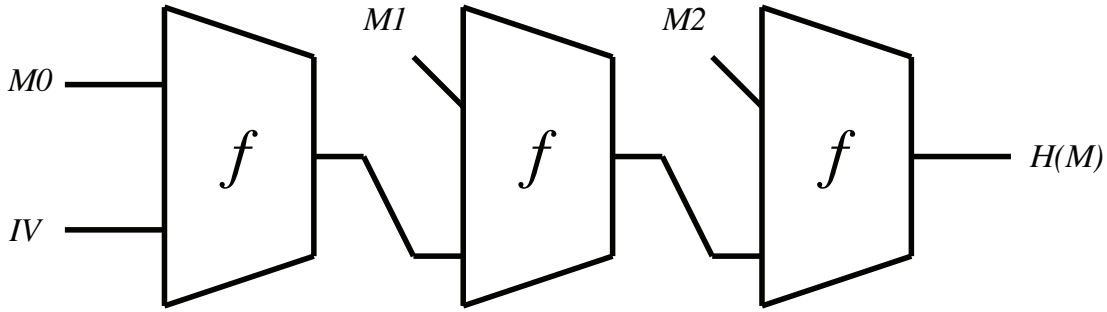
- Find a collision for this new function H'. 50% of the time, the two values will be one based on G_1 and one on G_2

Forging signatures



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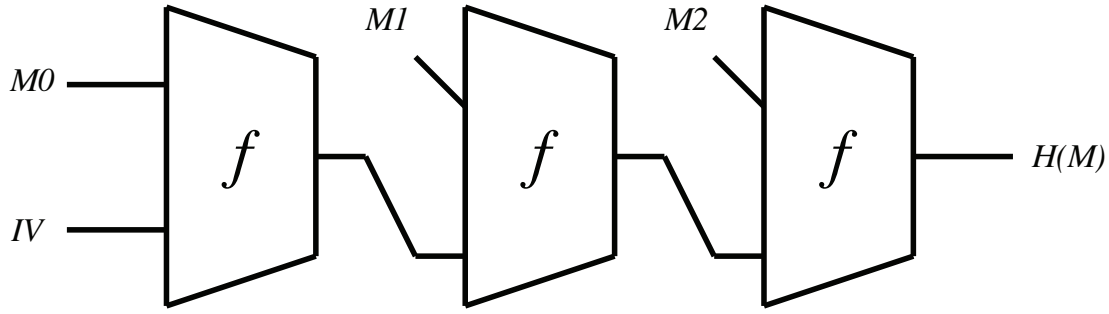

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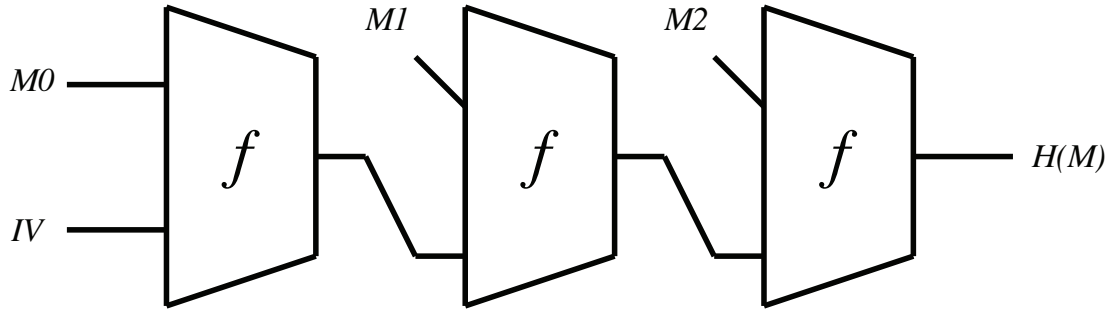

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- So if for two different messages, the output at a specific stage is the same, the output from then onward is the same, if the message from then onward is the same.

Thank you!

Some references: (Well, one, anyway)

- Most work on attacking the MDx and SHA family of hashes was done by Dr. Xiaoyun Wang. There are several papers describing the attacks on her website: `http://www.infosec.sdu.edu.cn/people/wangxiaoyun.htm`